Volume 4, No. 8 August 2025
p-ISSN 2980-4868 | e-ISSN 2980-4841
https://ajesh.ph/index.php/gp

Developing a Mathematical Model for Training Center Timetabling Problem: A Case Study

Susilowati, Nur Aini Masruroh
Gadjah Mada University, Indonesia
Email: susilowati1994@mail.ugm.ac.id, aini@ugm.ac.id

ABSTRACT
This study develops a mathematical model to address the multi-location training center timetabling problem, focusing on the Yogyakarta Industrial Training Center. Effective scheduling is crucial for optimizing resource utilization, minimizing conflicts, and ensuring equitable workload distribution among staff. The research identifies the challenges faced by conventional scheduling methods and proposes an Integer Programming approach to enhance scheduling efficiency. By analyzing historical workload data and conducting sensitivity analyses, the study demonstrates the model's effectiveness in achieving balanced workloads and reducing scheduling conflicts. The findings aim to improve operational efficiency and participant satisfaction in training programs.
Keywords: Training center, timetabling problem, mathematical model, Integer Programming, workload distribution, scheduling efficiency.

INTRODUCTION

The scheduling of activities in training institutions plays a crucial role in ensuring the effectiveness and efficiency of the implementation of training programs. Optimal scheduling not only impacts the smooth operation of the institution, but also affects the quality of training results and participant satisfaction. The main goal of scheduling is to optimize one or more goals, such as resource utilization, conflict minimization, and equitable distribution of workloads (Urva & Sellyana, 2019). Without good scheduling, training institutions can face various problems, such as workload imbalances, time conflicts, and inefficiencies in the use of human resources.

The process of scheduling work teams at training institutions is not simple. This scheduling process involves a variety of dynamic variables and constraints, including the type of training, location, duration of training, and the availability of human resources from various roles, such as teachers, coordinators, and support staff. Complexity is heightened when it must be ensured that there is no overlap of schedules and workloads are distributed fairly and evenly among team members. In addition, external factors such as sudden changes in training or policy requests from agencies or governments often add to the challenge. Without proper handling, this risks affecting the performance and quality of the training held (Nugraha, 2020).

Conventional scheduling methods, such as manual or spreadsheet-based approaches, are often ineffective at handling the scale and complexity of scheduling in large, dynamic training institutions. The use of this method often results in inefficient schedules, takes time in preparation, causes schedule conflicts between personnel, and distributes workloads unevenly. This inefficiency has the potential to disrupt the quality of training and overall team performance (Nugroho, 2020). Therefore, a more sophisticated and adaptive approach is needed to overcome this problem.

Modeling with Integer Programming provides a powerful mathematical framework for optimally formulating scheduling problems with measurable constraints. From the mathematical model that has been built, it is possible to find solutions using exact methods, metaheuristic methods (genetic algorithms, simulated annealing, tabu search, etc.) or using hybrid methods depending on the complexity of the model that has been built. The exact method is used to explore all possible solutions to problems with small or medium sizes (Wirawan, 2017). The metaheuristic method is used for the search for near-optimal solutions in large-scale cases (Santoso & Ai, 2017). In addition, there are hybrid methods that combine several metaheuristic methods, also offering more efficient solutions to deal with complex scheduling problems (Priambodo et al., 2016). These approaches provide advantages in model flexibility, computational efficiency, and the ability to overcome various limitations when compared to manual (conventional) scheduling methods.

The Yogyakarta Industrial Education and Training Center (BDI Yogyakarta) is a technical implementation unit in the field of industrial education and training which is under and responsible to the Head of the Industrial Education and Training Center (Pusdiklat Industri) of the Ministry of Industry. The training conducted by BDI Yogyakarta uses a 3-in-1 system, namely training, competency certification and job placement in one flow of activities. As one of the industrial training institutions in Indonesia, BDI Yogyakarta faces challenges in managing training programs that are increasing every year. Based on the Monitoring and Evaluation Report of the Yogyakarta Industrial Training Center, in 2020 this institution managed 89 training batches with 6 types of training, and in 2021 this number increased sharply to 214 batches with 10 types of training, there was an increase of 104% triggered by the implementation of online training during the COVID-19 pandemic. In 2022 and 2023, the number of training reached 116 and 120 batches, respectively, with 6 types of training programs in 2022 and 11 types of training programs in 2023. Meanwhile, in 2024, the number of trainings will be 104 trainings with 9 types of training programs. (Yogyakarta Industrial Education and Training Center, 2024).

With the increase in demand for the number of trainings, time constraints and limited human resources, optimal scheduling is an urgent need. On average, the preparation of the work team schedule every month requires a long process and time (± 5 working days), so that the preparation made by the personnel (committee) before the training is carried out is quite short. This can have an impact on the quality of the training provided and the satisfaction of the participants. Based on the 2024 Monitoring and Evaluation Report, participants' satisfaction with the committee's performance has decreased in the last two years, namely in 2023 and 2024. In 2023 there was a decrease of 0.32% and in 2024 there was a decrease of 1.8%. If the decline in participant satisfaction with the performance of this committee is left unchecked, it can have a negative impact on various other aspects, such as the reputation of the institution, stakeholder trust and a decrease in participant participation.

Full Time Equivalent (FTE) is a time-based workload analysis method by measuring the length of time it takes to complete work and then convert that time into the FTE value index. The FTE calculation can also be done by comparing the total employee workload with the target workload. This approach is often used in workload analysis to measure efficiency and determine human resource needs. The implications of FTE values are divided into 3 types, namely overload, normal, and underload (Tridoyo and Sriyanto, 2014). Based on the workload analysis guidelines issued by the State Civil Service Agency in 2010, the total value of the FTE index that is above the value of 1.28 is considered overloaded, between values 1 to 1.28 is considered normal, while if the value of the FTE index is between 0 and 0.99, it is considered underload or the workload is still lacking.

Based on data from the Employee Performance Recap of the Yogyakarta Industrial Training Center in 2024, there is a difference in workload between personnel in each division. Table 1.1 presents workload data for the training coordinator division, Table 1.2 presents workload data for the teaching division, Table 1.3 presents workload data for the monitoring division, and Table 1.4 presents workload data for the finance division.

Table 1.1 Workload Data for the Coordinating Division (Yogyakarta Industrial Training Center, 2024)

Coordinator	Expected Number of Training	Expected Days Worked	Number of Training	Days Worked	FTE Index	Note
K1	11	22	10	20	0,91	Underload
K2	11	22	7	14	0,64	Underload
K3	11	22	15	30	1,36	Overload
K4	11	22	16	32	1,45	Overload
K5	11	22	10	20	0,91	Underload
K6	11	22	11	22	1,00	Normal/Fit
K7	11	22	10	20	0,91	Underload
K8	11	22	9	18	0,82	Underload
K9	11	22	16	32	1,45	Overload
TOTAL			**104**			

Table 1.2 Workload Data for the Teaching Division (Yogyakarta Industrial Training Center, 2024)

Instructor	Expected Number of Training	Expected Days Worked	Number of Training	Days Worked	FTE Index	Note
P1	14	28	10	20	0,71	Underload
P2	14	28	19	38	1,36	Overload
P3	14	28	20	40	1,43	Overload
P4	14	28	12	24	0,86	Underload
P5	14	28	20	40	1,43	Overload
P6	14	28	12	24	0,86	Underload
P7	14	28	11	22	0,79	Underload
TOTAL			**104**			

Table 1.3 Workload Data for the Monitoring Division (Yogyakarta Industrial Training Center, 2024)

Monitoring Team	Expected Number of Training	Expected Days Worked	Number of Training	Days Worked	FTE Index	Note
TM1	6	12	6	12	1,00	Normal/Fit
TM2	6	12	14	28	2,33	Overload
TM3	6	12	6	12	1,00	Normal/Fit
TM4	6	12	8	16	1,33	Overload
TM5	6	12	10	20	1,67	Overload
TM6	6	12	9	18	1,50	Overload
TM7	6	12	10	20	1,67	Overload
TM8	6	12	4	8	0,67	Underload
TM9	6	12	6	12	1,00	Normal/Fit
TM10	6	12	7	14	1,17	Normal/Fit
TM11	6	12	6	12	1,00	Normal/Fit
TM12	6	12	5	10	0,83	Underload
TM13	6	12	5	10	0,83	Underload
TM14	6	12	4	8	0,67	Underload
TM15	6	12	4	8	0,67	Underload
TOTAL			**104**			

Table 1.4 Workload Data for Finance Division (Yogyakarta Industrial Training Center, 2024)

Finance Team	Expected Number of Training	Expected Days Worked	Number of Training	Days Worked	FTE Index	Note
F1	13	26	14	28	1,08	Normal/Fit
F2	13	26	12	24	0,92	Underload
F3	13	26	13	26	1,00	Normal/Fit
F4	13	26	19	38	1,46	Overload
F5	13	26	13	26	1,00	Normal/Fit
F6	13	26	21	42	1,62	Overload
F7	13	26	6	12	0,46	Underload
F8	13	26	6	12	0,46	Underload
TOTAL			**104**			

Based on the data in Table 1.1, Table 1.2, Table 1.3 and Table 1.4, there will be an uneven distribution of workload among personnel in each division in 2024. For the coordinating division, the percentage of employees with the low workload category (underload) is 55.6%, the normal category is 11.1% and the overload category is 33.3%. For the teaching division, the percentage of employees with the low workload category (underload) is 57.1%, the normal category is 0% and the overload category is 42.9%. Then for the monitoring division, the percentage of employees with the low workload category (underload) is 33.3%, the normal category is 33.3% and the overload category is 33.3%. As for the finance division, the percentage of employees with the low workload category (underload) is 37.5%, the normal category is 37.5% and the overload category is 25%. The gap between the highest and lowest workloads is still quite wide, so improvements need to be made in the distribution of workloads.

Comparison of workload data for employees of the Yogyakarta Industrial Education and Training Center from 2021 to 2024 is presented in Table 1.5.

Table 1.5 Employee Workload from 2021 to 2024 (Yogyakarta Industrial Training Center, 2024)

Coordinator				
Year	**2021**	**2022**	**2023**	**2024**
Underload	41,7%	45,5%	50,0%	55,6%
Normal/Fit	8,3%	9,1%	8,3%	11,1%
Overload	50,0%	45,5%	41,7%	33,3%
Instructor				
Year	**2021**	**2022**	**2023**	**2024**
Underload	62,5%	50,0%	57,1%	57,1%
Normal/Fit	0,0%	0,0%	14,3%	0,0%
Overload	37,5%	50,0%	42,9%	42,9%
Monitoring Team				

Year	2021	2022	2023	2024
Underload	50,0%	50,0%	50,0%	33,3%
Normal/Fit	0,0%	0,0%	8,3%	33,3%
Overload	50,0%	50,0%	41,7%	33,3%
Finance Team				
Year	**2021**	**2022**	**2023**	**2024**
Underload	50,0%	33,3%	50,0%	37,5%
Normal/Fit	16,7%	33,3%	0,0%	37,5%
Overload	33,3%	33,3%	50,0%	25,0%

Based on Table 1.5, workload data for each division from 2021 to 2024 has the same trend, namely the number of employees with the normal/fit workload category is still much lower than the number of employees with the underload category and the overload category. Therefore, it is necessary to make improvements in the scheduling of the work team so that the distribution of workload can be more even and still in the category of normal/fit workload.

This research contributes to the development of mathematical models for more efficient and effective scheduling in the setting of training institutions. By modeling using Integer Programming and finding optimal solutions, this study is expected to offer better solutions in the context of equitable workload distribution and minimization of schedule conflicts. The results of this research are not only expected to be implemented at the Yogyakarta Industrial Training Center, but also have the potential to be applied at the Industrial Training Center and other training institutions that face similar challenges. In addition, this research can be a foothold for the development of automated scheduling systems in the future that are more sophisticated and responsive to changing operational needs.

In the practice of scheduling work teams at the Yogyakarta Industrial Training Center, the process is still manual using a regular spreadsheet, without utilizing solvers or special applications for automatic scheduling. This situation causes scheduling to be complicated and complex, where the resulting schedule results do not pay attention to the distribution of workloads evenly. As a result, there is a risk of work inefficiency, either due to a workload that is too heavy or too light. These inefficiencies can disrupt the quality of training and customer satisfaction, which includes industry, participants, and other external parties, and potentially harm the team's overall performance. Therefore, the development of mathematical models using Integer Programming and the search for optimal solutions in scheduling are expected to overcome these problems. This study aims to develop an optimal mathematical model of work team scheduling by considering several factors, such as the availability of human resources, the number of trainings, the location, and the duration of training. In addition, this study also aims to analyze the effectiveness of the developed model in solving the problem of uneven distribution of workloads, as well as finding optimal solutions to complex scheduling problems more effectively and efficiently. Through the comparison of optimization results with conventional scheduling methods previously applied, it is hoped that it can provide recommendations for improving the automatic scheduling system at the Yogyakarta Industrial Training Center and other training institutions. The benefits of this research include theoretical contributions in the development of theories in the field of scheduling optimization, as well as practical benefits for training institutions and decision-makers. The results of this research are expected to be directly applied at the Yogyakarta Industrial Training Center and adopted by other training institutions to optimize work team scheduling, with the hope of improving operational efficiency and a fairer distribution of workload among team members, as well as maintaining or even improving the quality of training and participant satisfaction. For decision-makers, this study provides useful information and models to optimize the use of human resources, so as to reduce time and costs in the complex scheduling process.

METHOD

In this study, the object of the research is the Yogyakarta Industrial Education and Training Center, which is a work unit under the Ministry of Industry of the Republic of Indonesia. The Yogyakarta Industrial Training Center has the main task of organizing 3-in-1 system-based training.

Research Stages

The stages in this study are presented in Figure 4.3.

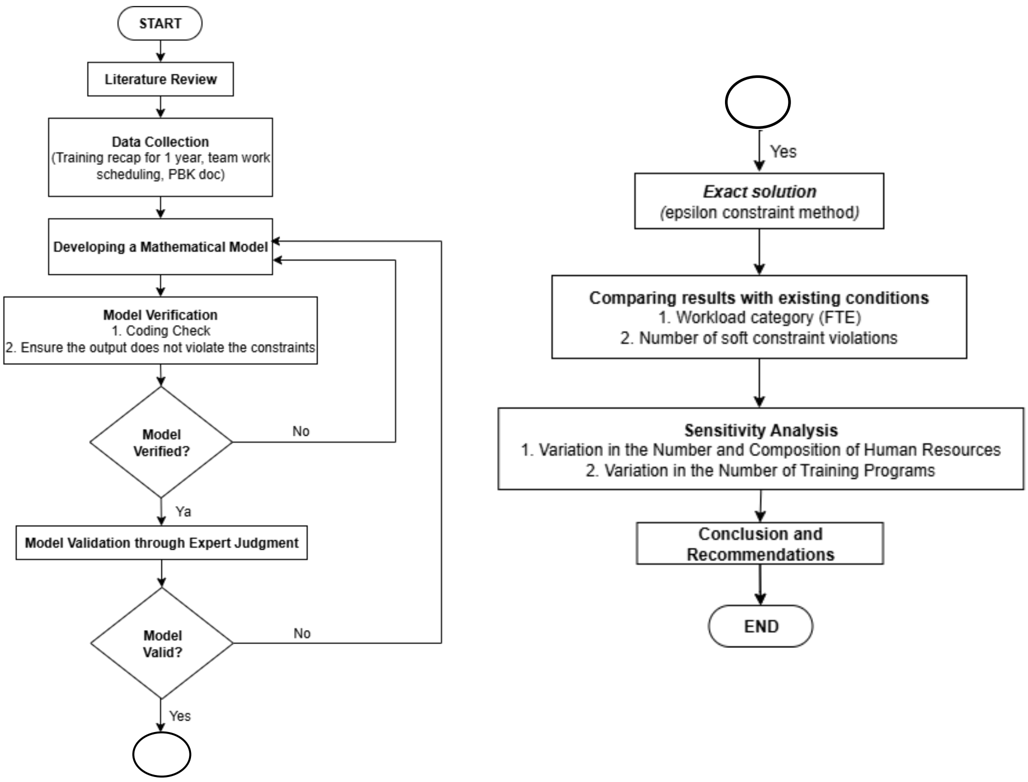


Figure 4.3 Research Stages

After the mathematical model is verified and declared valid, the next stage is to find the optimal solution for the scheduling of the work team according to the objectives of this study. The search for a solution was carried out using the exact method, namely the epsilon constraint method and solved using the Python Solver Gurobi. If the exact method is not able to produce an optimal solution with feasible computational time, then a solution search is carried out using the Metaheuistic method.

RESULT AND DISCUSSION

5.2 Mathematical Model of Work Team Scheduling

In this study, a mathematical model was developed to make the scheduling of the work team in accordance with the real system at the Yogyakarta Industrial Training Center. In this study, the function of the objectives to be achieved is the minimization of workload deviation between personnel in each division as shown in equation 5.1 and the minimum of total penalties due to violations of location, schedule clashes and workload in 1 month as shown in equation 5.2. The set of indices and parameters is presented in Table 5.4, while for Variables is presented in Table 5.5.

Table 5.4 Index Sets and Parameters

Index	Description
E	Training Set, with $e = 1,2,\dots,n_e$
D	Division Set, with $d = 1,2,3,4$ $(1 = \text{instructor}, 2 = \text{coordinator}, 3 = \text{monitoring}, 4 = \text{finance})$
I_d	Personnel Set in division d
M	Month Set, with $m = 1,2,\dots,12$
duration_e	Training duration e (days)
type_e	Types of training e
location_e	Location of training e
start_date_e	Start date of training e
end_date_e	End date of training e
n_d	Number of personnel in division d
n_e	Number of training e
ε	Penalty limit value
T_{pk_e}	Arrival time of division 1 and division 2 for training e
T_{m_e}	Arrival time of division 3 at training e
T_{f_e}	Arrival time of division 4 for training e
Max_Training	Maximum training limit per month per personnel

Table 5.5 Variable

Independent Variables	Description
$X_{i,d,e}$	$= 1$, if personnel i from division d is assigned for training e $= 0$, othewise

Dependent Variable	Description
δ_{id}	absolute difference workload W_{id} from average workload in division d (avg_d)
W_{id}	Total number of training e assigned to personnel i in division d
avg_d	Average workload in division d
FTE	Full Time Equivalent
$p\_bentrok_{i,d,e}$	$= 1$, if personnel i from division d is assigned for training e dan e + 1 with the same time $= 0$, otherwise
$p\_max_{i,d,m}$	$= 1$, if personnel i from division d is assigned more than $Max_{Training}$ in month m $= 0$, otherwise
P	Total penalty/Violation

The model developed is in the form of Mixed-Integer Programming with multi-objective. Equations 5.1 and 5.2 are the objective functions of the proposed MIP model. The first objective function is to minimize the deviation of workload between personnel in each division and the second objective function is to minimize the total penalty/violation. The elaboration of objective functions is described in equations 5.1.1 to 5.2.1.

$\text{Minimize } F_1 = \delta_{id}$ (5.1)

$\text{Minimize } F_2 = P$ (5.2)

The value of δ_{id} is the result of the workload of personnel i in division d () minus the average workload in division d ().$W_{id} avg_d$

$\delta_{id} = |W_{id} - avg_d|$ (5.1.1)

The value of W_{id} is the total training assigned to personnel i in division d

$$W_{id} = \sum_{e \in E} X_{i,d,e}, \quad \forall i \in I_d, \forall d \in D$$

(5.1.2)

Meanwhile, the value of avg_d is the result of adding up the workload of each personnel in the division and then dividing it by the number of personnel in the division.

$$avg_d = \frac{\sum_{i \in Id} W_{id}}{n_d}, \quad \forall d \in D$$

(5.1.3)

The workload category is calculated using the FTE (*Full Time Equivalent*) value, where the FTE calculation can be done by comparing the total workload of personnel with the target workload. Based on the workload analysis guidelines issued by the State Civil Service Agency in 2010, the total value of the FTE index that is above the value of 1.28 is considered *overloaded*, between values 1 to 1.28 is considered *normal*, while if the value of the FTE index is between 0 and 0.99, it is *considered underload* or the workload is still lacking.

$$\mathrm{FTE} = \frac{W_{id}}{\text{Target beban kerja}} = \frac{W_{id}}{\frac{n_e}{n_d}}$$

(5.1.4)

The total penalty/violation is calculated by adding up the total location penalty, the total schedule clash penalty and the total penalty against the maximum amount of training that can be handled by each personnel in 1 month.

$$P = \sum_{d \in D} \sum_{i \in I_d} \sum_{e \in E} \text{p\_lokasi}_{i,d,e} + \sum_{d \in D} \sum_{i \in I_d} \sum_{e \in E} \text{p\_bentrok}_{i,d,e} + \sum_{d \in D} \sum_{i \in I_d} \sum_{e \in E} \text{p\_max}_{i,d,m}$$

(5.2.1)

Equations 5.3 to 5.15 are the limitations used in this study and are a development of the model from the research of Mokhtari *et al.* (2021) and Rosyidi *et al.* (2019) which has been adapted to the real system at the Yogyakarta Industrial Training Center.

a. Equation 5.3 states that the training completion date is calculated from the training start date and the training duration.

$$\text{end\_date}_{\,e} = \text{start\_date}_{\,e} + \text{duration}_e - 1 \quad (5.3)$$

b. Equation 5.4 states that the arrival date of division 1 (teacher) and division 2 (coordinator) to each training location is on the first 2 (two) days.

$$T_{pk\_e} = \text{start\_date}_{\,e} \text{ dan } (\text{start\_date}_{\,e} + 1) \quad (5.4)$$

c. Equation 5.5 states that the arrival date of division 3 (monitoring team) to each training location is 2 (two) days in the middle of the training.

$$T_{m\_e} = (\text{start\_date}_{\,e} + \frac{\text{duration}_e}{2}) \text{ dan } (\text{start\_date}_{\,e} + \frac{\text{duration}_e}{2} + 1 \quad (5.5)$$

d. Equation 5.6 states that the arrival date of division 4 (finance team) to each training location on the last 2 (two) days of training.

$$T_{f\_e} = (\text{end\_date}_{\,e} - 1) \text{ dan end\_date}_{\,e} \quad (5.6)$$

e. Equation 5.7 states that each training requires 1 (one) personnel from each division.

$$\sum_{i \in I_d} X_{i,d,e} = 1, \quad \forall d \in D, \quad \forall e \in E \quad (5.7)$$

f. Equation 5.8 states that teaching personnel (division 1) come to each training location on the first 2 (two) days of training.

$$\sum_{t \in T_{pk\_e}} X_{i,d,e} = 1, \quad \forall i \in I_1, \forall e \in E \quad (5.8)$$

g. Equation 5.9 states that the coordinating personnel (division 2) come to each training location on the first 2 (two) days of training.

$$\sum_{t\in T_{pk\_e}} X_{i,d,e} = 1, \quad \forall i \in I_2, \forall e \in E$$

(5.9)

h. Equation 5.10 states that personnel from the monitoring team (division 3) come to each training location on 2 (two) days in the middle of the training.

$$\sum_{t\in T_{m\_e}} X_{i,d,e} = 1, \quad \forall i \in I_3, \forall e \in E$$

(5.10)

i. Equation 5.11 states that personnel from the finance team (division 4) come to each training location on the last 2 (two) days of training.

$$\sum_{t\in T_{f\_e}} X_{i,d,e} = 1, \quad \forall i \in I_4, \forall e \in E$$

(5.11)

j. Equation 5.12 states that each personnel shall not conduct training at the same location in succession.

$$X_{i,d,e} + X_{i,d,e+1} \leq 1 + p\_lokasi_{i,d,e}, \quad \forall d \in D, \forall i \in I_d, \forall e \in E$$
$$\text{with location}_{e} = \text{location}_{e+1}$$

(5.12)

k. Equation 5.13 states that each personnel can handle training in parallel as long as there is no time conflict.

$$X_{i,d,e} + X_{i,d,e+1} \leq 1 + M.\ p\_bentrok_{i,d,e}, \quad \forall i \in I_d, \forall d \in D, \forall e_1, e_2 \in E, e_1 \neq e_2$$
$$\text{jika ada bentrok jadwal}$$

(5.13)

l. Equation 5.14 states that each personnel must not handle more than the maximum amount of training set (*Max_Training*) in 1 (one) month.

$$\sum_{e\in E_m} X_{i,d,e} \leq Max_{Training} + M.p\_max_{i,d,m}, \quad \forall d \in D, \forall i \in I_d, \forall m \in \{1,2,\ldots,12\}$$

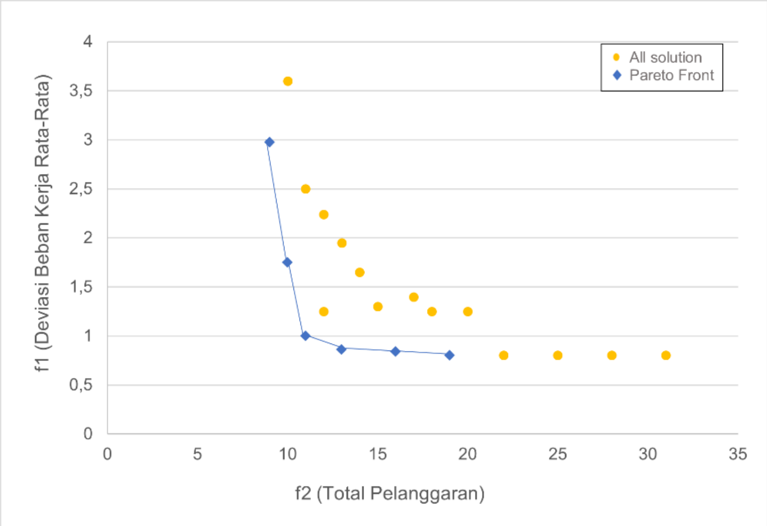
(5.14)

m. Equation 5.15 states the total penalty limit.

$$\sum_{d\in D}\sum_{i\in I_d}\sum_{e\in E} p\_lokasi_{i,d,e} + \sum_{d\in D}\sum_{i\in I_d}\sum_{e\in E} p\_bentrok_{i,d,e} + \sum_{d\in D}\sum_{i\in I_d}\sum_{e\in E} p\_max_{i,d,m} \leq \varepsilon$$

(5.15)

The model contains two objective functions, namely minimizing workload deviations between personnel in each division and minimizing total penalties/violations. To solve the multiobjective problem, *the epsilon constraint* method is used. The selection of this method was based on the results of the initial trial using 2024 training data. The test resulted in *a Pareto Front* diagram that showed a significant *trade-off* between the two optimized objectives, namely minimizing workload deviation (f1) and minimizing the number of violations (f2). The *Pareto Front diagram* is presented in Figure 5.1.



Gambar 5.1 Diagram *Pareto Front*

Based on this visualization, it can be seen that there is no one solution that is absolutely better in both aspects at once. Each Pareto solution has advantages on one goal, but at the expense of another. Therefore, the use of the ε-constraint method is considered appropriate because it allows the exploration of various Pareto solutions by maintaining one objective as the main function that is minimized, while the other goal is treated as constraints with varying ε limits. This approach provides flexibility in producing optimal solutions that match the scheduling policy priorities at the relevant institution.

Based on the model that has been built, the main objective function chosen is to minimize the deviation of workload between personnel in each division. This is because the main goal of developing this model is to level the workload between personnel. Meanwhile, the second objective function, which is to minimize the total penalty/offense, will be limited by the epsilon value as described in Equation 5.15.

The completion of the model was carried out using Solver Gurobi version 12.0.1 which is applied with the Python programming language on laptops with Intel Core-i5, 1.80 GHz CPU and 8.00 GB of RAM.

5.3 Model Verification and Validation

Model verification is used to ensure that the model is properly constructed and functioning according to established mathematical specifications or formulations. The goal is to ensure that mathematical models created and translated into the Python programming language can run without errors and that their output does not violate the established constraints. The verification process is carried out by checking the *coding (syntax)* by running the model and also checking the results manually. The running model is shown in Figure 5.2.

```
Academic license 2622689 - for non-commercial use only - registered to su   @gmail.com
Optimize a model with 14055 rows, 17655 columns and 65616 nonzeros
Model fingerprint: 0x6a92f410
Variable types: 43 continuous, 17612 integer (17573 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+02]
  Objective range  [7e-02, 1e-01]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 5e+01]
Presolve removed 351 rows and 351 columns
Presolve time: 0.13s
Presolved: 13704 rows, 17304 columns, 64017 nonzeros
Variable types: 0 continuous, 17304 integer (17222 binary)

Root relaxation: objective 0.000000e+00, 1808 iterations, 0.10 seconds (0.07 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0    0.00000    0  104          -    0.00000      -     -    0s
     0     0    0.30849    0  277          -    0.30849      -     -    0s
     0     0    0.33592    0  261          -    0.33592      -     -    2s
     0     0    0.74812    0  181          -    0.74812      -     -    2s
     0     0    0.77728    0  172          -    0.77728      -     -    2s
     0     0    0.79956    0  210          -    0.79956      -     -    2s
     0     0    0.86317    0  132          -    0.86317      -     -    2s
     0     0    0.86317    0  158          -    0.86317      -     -    3s
     0     0    0.86317    0  127          -    0.86317      -     -    3s
     0     0    0.86317    0  150          -    0.86317      -     -    3s
     0     0    0.86317    0  107          -    0.86317      -     -    3s
     0     0    0.86317    0   40          -    0.86317      -     -    4s
     0     0    0.86317    0   39          -    0.86317      -     -    4s
     0     0    0.86317    0   54          -    0.86317      -     -    4s
     0     0    0.86317    0   71          -    0.86317      -     -    5s
     0     0    0.86317    0   36          -    0.86317      -     -    5s
     0     0    0.86317    0   36          -    0.86317      -     -    5s
     0     0    0.86317    0   27          -    0.86317      -     -    6s
     0     0    0.86317    0   22          -    0.86317      -     -    6s
     0     0    0.86317    0   45          -    0.86317      -     -    6s
     0     0    0.86317    0   45          -    0.86317      -     -    7s
H    0     0                       0.8631696    0.86317  0.00%     -    7s
     0     0    0.86317    0   45    0.86317    0.86317  0.00%     -    7s

Cutting planes:
  Gomory: 33
  Cover: 4
  MIR: 27
  Zero half: 12
  RLT: 14
  BQP: 46

Explored 1 nodes (30795 simplex iterations) in 7.70 seconds (6.20 work units)
Thread count was 2 (of 2 available processors)

Solution count 1: 0.86317

Optimal solution found (tolerance 1.00e-04)
Best objective 8.631695641219e-01, best bound 8.631695641219e-01, gap 0.0000%
Solusi optimal ditemukan.
```

Gambar 5.2 Running Mode

The next process is to check whether the output displayed is in accordance with the restrictions that have been made, by checking directly on the output scheduling results. Based on manual checking, there are no hard constraints violated and the total violation of the soft constraint that occurs does not exceed the set epsilon value. Therefore, the model can be declared verified.

Model validation is used to ensure that the model is substantively correct and accurately describing the real-world phenomenon or system being modeled. In other words, validation focuses on how well the model is able to represent reality and provide relevant and useful results. The validation of this model is also carried out through expert judgment.

5.4 Scheduling Results

The results of the work team scheduling model can be seen in Appendix 5. The model is solved by the epsilon constraint method, where the main goal function is to minimize the deviation of workload between personnel in each division, while the second goal function is to minimize the total penalty/violation is limited by the epsilon value. The initial epsilon value is set at 10, which means that the solution is only accepted if the number of violations does not exceed 10. This value is assumed from the maximum violation tolerance limit of 1 violation per month, taking into account the number of effective months in a year is 10 months. If no feasible solution is found at this limit, the epsilon value will be gradually increased until a solution that meets the limit is obtained. This approach aims to maintain a balance between a fair distribution of workload and minimizing constraint violations, especially when the number of human resources is limited. The model is run using the Gurobi solver with the Python programming language, with a branch and bound solver type. The model is run for training data from 2021 to 2024. From the results of the scheduling, the results of optimizing the distribution of workload and total violations for the 2021 to 2024 data as shown in Table 5.7 were obtained.

Table 5.7 Recap of Optimization Results for 2021 to 2024

Year	Workload (FTE)			Number of Violations			
	Underload	Normal	Overload	Location Constraints	Schedule Conflict Constraints	Constraint Max_Training	Total
2021	0%	100%	0%	0	0	10	10
2022	0%	100%	0%	0	2	2	4
2023	0%	100%	0%	0	0	0	0
2024	0%	100%	0%	0	1	9	10

Based on Table 5.7, the deviation of workload between personnel can be minimized by the model, even consistently from 2021 to 2024, the workload of all employees is in the Normal/Fit category, without any underload or overload. While there are still violations, the model has managed to reduce the number of violations and minimize overassignment compared to no-optimization scenarios.

5.5. Comparison of Scheduling Results

To analyze the effectiveness of the developed model, a comparison was made between the results of scheduling using a mathematical model and the results of scheduling under actual conditions without optimization. This comparison includes aspects of workload distribution as well as the number of violations against various existing constraints. Table 5.16 presents a comparison of the results of the reorganization from 2021 to 2024.

Table 5.16 Comparison of Model vs Actual Scheduling Results

Year		Workload (FTE)			Number of Violations			
		Underload	Normal	Overload	Location Constraints	Schedule Conflict Constraints	Constraint Max_Training	Total
2021	Model	0%	100%	0%	0	0	10	10
	Current	50,0%	5,6%	44,4%	64	40	12	114
2022	Model	0%	100%	0%	0	2	2	4
	Current	45,9%	8,1%	45,9%	2	8	3	13
2023	Model	0%	100%	0%	0	0	0	0
	Current	51,4%	5,4%	43,2%	2	18	3	23
2024	Model	0%	100%	0%	0	1	9	10
	Current	43,6%	23,1%	33,3%	4	42	6	52

Table 5.16 compares workload and number of violations between mathematical (optimization) and actual (non-optimization) models over four years, from 2021 to 2024. The evaluation was carried out based on workload (FTE) and the number of violations against three types of constraints, namely location constraints, schedule clashes, and maximum training limits.

1. Workload based on Full Time Equivalent (FTE) calculations

   From the results of the comparison, the mathematical model succeeded in distributing the workload more evenly compared to the actual conditions. In the model, all personnel are in the category of normal workload (100%), with no one experiencing underload or overload. On the other hand, in actual conditions, there is a fairly high proportion of personnel with a fairly high underload (43% - 52%), while some others experience significant overload (33% - 46%). This shows that without optimization, there is an imbalance in the allocation of tasks that can result in a decrease in work efficiency.
2. Number of constraint violations

   The three main obstacles analyzed were location, schedule clashes, and maximum training limits. In general, optimization models result in a much lower number of violations than actual conditions. This shows that the optimization system has managed to minimize location-related conflicts, schedule conflicts, and maximum training limitations and indicates that the resulting solution is more feasible against the set rules.

Based on the results of this comparison, the use of the optimization model in work team scheduling is highly recommended, as it has been proven to improve workload balance and minimize schedule conflicts and rule violations.

5.6 Sensitivity Analysis

The function of sensitivity analysis is to understand how changes in the input parameters of a model affect the output of that model. The first sensitivity analysis in this study was conducted to see how changes in the number of human resources in each division affect the results of the work team's scheduling. A second sensitivity analysis was conducted to see how strong the model maintains workload balance as the number of trainings increases.

5.6.1 Total HR Scenario

By trying various scenarios for the number of human resources, we can find out if the schedule made is still running well, as well as see if there is a violation of the set rules. The results of this analysis help in determining the ideal number of human resources so that the workload remains balanced and schedule conflicts do not occur. The sensitivity analysis related to the number of human resources was tested through three scenarios, namely:

1. The number of human resources in each division varies ± 2 people,
2. The number of human resources is evenly distributed in all divisions,
3. The number of human resources is evenly distributed except in the finance division.

The results of sensitivity analysis with various scenarios of the number of human resources are shown in Tables 5.17, 5.18 and 5.19.

Table 5.17 Sensitivity Analysis of the Number of Human Resources Scenario 1

Year	Number of human resources					Workload (FTE)			Number of Violations			
	Division 1	Division 2	Division 3	Division 4	Total	Under load	Normal	Over load	Location Constraints	Constraint Bentrok	Constraint Max_ Training	Total
2021	6	10	8	4	28	0%	100%	0%	0	25	25	50
	7	11	9	5	32	0%	100%	0%	0	3	17	20
	8	12	10	6	36	0%	100%	0%	0	0	10	10
	9	13	11	7	40	0%	100%	0%	0	0	7	7
	10	14	12	8	44	0%	100%	0%	0	0	3	3
2022	8	9	8	4	29	0%	100%	0%	0	3	7	10
	9	10	9	5	33	0%	100%	0%	0	3	5	8
	10	11	10	6	37	0%	100%	0%	0	2	2	4
	11	12	11	7	41	0%	100%	0%	0	1	2	3
	12	13	12	8	45	0%	100%	0%	0	0	0	0
2023	5	10	10	4	29	0%	100%	0%	0	8	2	10
	6	11	11	5	33	0%	100%	0%	0	4	0	4
	7	12	12	6	37	0%	100%	0%	0	0	0	0
	8	13	13	7	41	0%	100%	0%	0	0	0	0
	9	14	14	8	45	0%	100%	0%	0	0	0	0

Year	Number of human resources					Workload (FTE)			Number of Violations			
	Division 1	Division 2	Division 3	Division 4	Total	Under load	Normal	Over load	Location Constraints	Constraint Bentrok	Constraint Max_ Training	Total
2024	5	7	13	6	31	0%	100%	0%	0	13	12	25
	6	8	14	7	35	0%	100%	0%	0	0	15	15
	7	9	15	8	39	0%	100%	0%	0	1	9	10
	8	10	16	9	43	0%	100%	0%	0	0	10	10
	9	11	17	10	47	0%	100%	0%	0	0	10	10

Table 5.18 Sensitivity Analysis of the Number of Human Resources Scenario 2

Year	Number of human resources					Workload (FTE)			Number of Violations			
	Division 1	Division 2	Division 3	Division 4	Total	Under load	Normal	Over load	Location Constraints	Constraint Bentrok	Constraint Max_ Training	Total
2021	7	7	7	7	28	0%	100%	0%	0	2	23	25
	8	8	8	8	32	0%	100%	0%	0	0	15	15
	9	9	9	9	36	0%	100%	0%	1	1	8	10
	10	10	10	10	40	0%	100%	0%	0	4	3	7
	11	11	11	11	44	0%	100%	0%	1	2	0	3
2022	7	7	7	7	28	0%	100%	0%	0	1	4	5
	8	8	8	8	32	0%	100%	0%	0	0	0	0
	9	9	9	9	36	0%	100%	0%	0	0	0	0
	10	10	10	10	40	0%	100%	0%	0	0	0	0
	11	11	11	11	44	0%	100%	0%	0	0	0	0
2023	7	7	7	7	28	0%	100%	0%	0	5	0	5
	8	8	8	8	32	0%	100%	0%	0	0	0	0
	9	9	9	9	36	0%	100%	0%	0	0	0	0
	10	10	10	10	40	0%	100%	0%	0	0	0	0
	11	11	11	11	44	0%	100%	0%	0	0	0	0
2024	8	8	8	8	32	0%	100%	0%	0	0	20	20
	9	9	9	9	36	8,33%	86,11%	5,56%	0	0	10	10
	10	10	10	10	40	0%	95,00%	5,00%	0	0	10	10
	11	11	11	11	44	0%	93,18%	6,82%	0	0	5	5
	12	12	12	12	48	0%	100%	0%	0	1	4	5

Table 5.19 Sensitivity Analysis of Number of Human Resources Scenario 3

Year	Number of human resources					Workload (FTE)			Number of Violations			
	Division 1	Division 2	Division 3	Division 4	Total	Under load	Normal	Over load	Location Constraints	Constraint Bentrok	Constraint Max_ Training	Total
2021	7	7	7	6	27	0%	100%	0%	0	1	29	30
	8	8	8	6	30	0%	100%	0%	0	1	14	15
	9	9	9	6	33	0%	100%	0%	0	1	11	12
	10	10	10	6	36	0%	100%	0%	0	1	9	10
	11	11	11	6	39	0%	100%	0%	0	0	7	7
2022	7	7	7	6	27	0%	100%	0%	0	1	4	5
	8	8	8	6	30	0%	100%	0%	0	2	1	3
	9	9	9	6	33	0%	100%	0%	0	2	0	2
	10	10	10	6	36	0%	100%	0%	0	0	1	1
	11	11	11	6	39	0%	100%	0%	0	0	1	1
2023	7	7	7	6	27	0%	100%	0%	1	2	2	5
	8	8	8	6	30	0%	100%	0%	0	0	0	0
	9	9	9	6	33	0%	100%	0%	0	0	0	0
	10	10	10	6	36	0%	100%	0%	0	0	0	0
	11	11	11	6	39	0%	100%	0%	0	0	0	0
2024	7	7	7	8	29	0%	100%	0%	0	7	21	30
	8	8	8	8	32	0%	100%	0%	0	0	20	20
	9	9	9	8	35	11,43%	82,86%	5,71%	0	0	10	10
	10	10	10	8	38	0%	94,74%	5,26%	0	0	10	10
	11	11	11	8	41	0%	100%	0%	0	0	10	10
	12	12	12	8	44	0%	100%	0%	0	0	7	7

Scenario 1: Number of human resources in each division ±2

The first scenario explores the change in the number of human resources in each division in the range of ±2 people. This aims to see how fluctuations in the number of human resources affect the distribution of workload and violations that occur. The main results of this scenario:

1. Regardless of the variation in the number of HR, all configurations indicate that the workload is in the 100% normal category. This means that the model is able to divide tasks evenly among available personnel, without underloading or overloading. This reflects the effectiveness of the model in managing workloads, even as the number of human resources changes.
2. Violations of constraints such as clashing training schedules and the maximum number of training per month for each personnel decrease as the number of human resources increases, suggesting that adding human resources provides greater scheduling flexibility and improves the ability to handle training spread across different times and locations.

3. The number of human resources that is too small (e.g. only 5-7 people per division) leads to a significant increase in violations. This is due to the limited combination of work teams that can be formed, so that schedule and workload conflicts cannot be avoided.

Scenario 2: The number of human resources is evenly distributed across all divisions

Scenario 2 is designed to observe the impact of equalizing the number of human resources across divisions, with equal numbers between Divisions 1, 2, 3, and 4. The goal is to evaluate whether an even HR allocation strategy between divisions is able to provide scheduling efficiency, workload balance, and minimize violations of constraints. The main results of this scenario:

1. In general, the workload in this scenario is in the 100% normal category on most configurations, especially when the total human resources are in the range of 36 to 44 people. In smaller HR formations (e.g. 8 people per division), there are slight deviations, such as the appearance of underload (8.33%) and overload (5.56%). However, when the number of human resources is even and large enough (≥10 per division), the distribution of workloads returns to stability, reaching 100% normal.
2. With an even allocation of human resources, the scheduling system has better flexibility to form work teams without having to rely excessively on specific divisions. This has an impact on reducing constraint violations, especially schedule clash violations and violations of the maximum training per month for each personnel.
3. The pattern that looks quite consistent: the higher the number of human resources per division (with an even distribution), the lower the number of violations. This shows that the existence of a backup of HR in each division increases the likelihood of team formation without schedule conflicts or overload.
4. Just like the previous scenario, the epsilon constraint method is applied with an epsilon initial value = 10, based on the assumption of a maximum of 1 violation per month in the 10 effective months per year. Most configurations in this scenario can meet the epsilon limit without the need to increase the value, especially in formations with a number of ≥9 human resources per division. This means that the strategy of equalizing the number of human resources between divisions is quite effective in producing feasible solutions without many violations.

Scenario 2 shows that an even HR allocation strategy between divisions has a positive impact on the stability of workload distribution and significantly reduces constraint violations. What's more, the scheduling system is able to accommodate training tasks efficiently without causing location conflicts or over-assignments. This strategy has also been proven to be in accordance with the epsilon constraint approach, because many feasible solutions are found without the need to raise the epsilon value from the initial limit.

Scenario 3: The number of human resources is evenly distributed, except for the finance division

Scenario 3 is designed to reflect real conditions, where the number of human resources in Division 4 (finance division) is relatively fixed each year. This is due to the specification of expertise in the financial sector that is not easily replaced by personnel from other divisions. Therefore, this sensitivity analysis evaluates the impact of increasing the number of human resources only in Divisions 1, 2, and 3, while the number of human resources in Division 4 is maintained at 6 or 8 people. The main outcomes of this scenario are:

1. Almost all configurations indicate that workloads are in the normal category, even with a limited number of human resources. Although certain formations such as 8-9-9-6 and 9-9-9-6 have workload deviations (underload 11.43% and overload up to 5.71%), most of them remain within good tolerance limits.
2. One of the key findings of this scenario is that the number of violations cannot be fully suppressed by simply adding human resources in Divisions 1–3. This can be seen from:
   a. The 9-9-9-6 formation still produces 30 violations, with the dominance on the maximum number of training constraints.
   b. In contrast, when Division 4 was also increased to 8 men (e.g. 10-10-10-8 formation), the offense decreased to 10, showing that Division 4's capacity also greatly determined the success of the scheduling.
3. The most violations come from the maximum limit on the number of training per individual. This happens because the limitation of human resources in Division 4 causes personnel in the division to be assigned more often, exceeding the maximum allowed limit. This did not happen much in the previous scenario that increased the number of human resources in a balanced manner in all divisions.
4. As in the previous scenario, the epsilon constraint approach is used with a starting limit of $\varepsilon = 10$. In this scenario, only configurations with Division 4 numbering ≥ 8 people meet the epsilon limit. Configurations with a 6-person Division 4 often exceed that limit, so it is necessary to raise epsilon to get a feasible solution. This confirms that the limitation of human resources in one critical division can be a dominant factor in the increase in system violations.

Scenario 3 shows that an increase in the number of human resources in three divisions alone (1, 2, and 3) is not enough to eliminate violations if Division

4 remains restricted. This is due to the limitations of the 4th Division's personnel making them too often scheduled, exceeding the maximum allowable task. Therefore, a more appropriate strategy would be to consider minimal upgrades in Division 4, especially when the amount of training and locations is large enough. The application of the epsilon constraint method in this scenario makes it clear that a feasible and optimal solution can only be obtained if all divisions, including divisions with special specializations, have adequate human resource allocation.

The Impact of Training Data Distribution

In addition to the number and distribution of human resources, the results of this sensitivity analysis are also greatly influenced by the pattern of distribution of training data itself. A tight amount of training in a given time or concentration of training in a specific location will add pressure to the scheduling system. For example, if there are many training sessions in one month that are of long duration and involve the same location, then the likelihood of schedule conflicts or location clashes will increase, especially if human resources are limited. Therefore, the availability of adequate human resources must be supported by the arrangement of training schedules that are evenly distributed in time and location.

From the overall sensitivity analysis related to the number of human resources, it can be concluded that:

1. The number of human resources greatly affects scheduling flexibility. The greater the number of human resources, the less likely it is to occur.
2. A balanced distribution between divisions is essential. Even though the total number of human resources is sufficient, the imbalance in the number of personnel between divisions can still cause problems.
3. Divisions with specific HR specifications such as finance require special attention, as limited personnel can be a critical point in the scheduling system even if other divisions are optimal.
4. The distribution of training data affects the system load. Scheduling will be more optimal if the training is evenly distributed in terms of time, location, and type of training.
5. The scheduling model is quite robust, because in most variations in the number of HR, scheduling results remain optimal with normal workloads and minimal breaches.

These results can be used as a reference in planning human resource needs for each period, as well as as a basis for strategic decision-making to maintain optimal operational performance.

5.6.2 Scenario Number of Training

This sensitivity analysis was conducted to evaluate the effect of variations in the number of training on HR allocation, workload distribution, and constraint violations, which include location violations, schedule clashes, and exceeding the maximum limit of training per individual. Based on the results of the sensitivity analysis to the number of human resources, especially in Scenario 3, it was found that the optimal human resource formation is one that is able to produce no more than 10 violations per year.

The formations that meet these criteria are 10-10-10-6 and 10-10-10-8, which means that they each consist of 10 people in Divisions 1, 2, and 3, as well as 6 or 8 people in Division 4. In addition to successfully keeping violations below the threshold, these two formations also show a workload distribution of more than 90% in the normal category, indicating that the division of tasks between personnel takes place fairly and proportionately. Based on this, the sensitivity analysis to the number of training is then focused on the following two scenarios:

1. Scenario 1: 10-10-10-6 HR formation, with variations in the number of training between 100 to 500 batches.
2. Scenario 2: 10-10-10-8 HR formation, with variations in the number of training between 100 to 500 batches.

These two scenarios aim to test the extent of the resilience of HR formations to the surge in the number of trainings, as well as to see the dynamics of changes in workload and the number of constraint violations as the volume of training activities increases. The results of the sensitivity analysis with the number of training scenarios can be seen in Tables 5.20 and 5.21.

Table 5.20 Sensitivity Analysis of Number of Training Scenarios 1

Number of Training	Number of human resources					Workload (FTE)			Number of Violations				Compute Time (Hours)
	Division 1	Division 2	Division 3	Divided 4	Total	Under load	Normal	Overload	Location Constraints	Constraint Bentrok	Constraint Max_ Training	Total	
100	10	10	10	6	36	0%	100%	0%	0	0	0	0	< 1
150	10	10	10	6	36	0%	100%	0%	0	0	0	0	
200	10	10	10	6	36	0%	100%	0%	0	0	0	0	
250	10	10	10	6	36	0%	100%	0%	0	0	0	0	
300	10	10	10	6	36	0%	100%	0%	0	2	3	5	
350	10	10	10	6	36	0%	100%	0%	0	2	8	10	3
400	10	10	10	6	36	0%	100%	0%	0	2	13	15	3.5
450	10	10	10	6	36	0%	100%	0%	0	7	18	25	4
500	10	10	10	6	36	0%	100%	0%	0	5	25	30	6

Table 5.21 Scenario 2 Training Number Sensitivity Analysis

Number of Training	Number of human resources					Workload (FTE)			Number of Violations				Compute Time (Hours)
	Divided 1	Divided 2	Divided 3	Divided 4	Total	Underload	Normal	Overload	Location Constraints	Constraint Bentrok	Constraint Max_ Training	Total	
100	10	10	10	8	38	0%	100%	0%	0	0	0	0	< 1
150	10	10	10	8	38	0%	100%	0%	0	0	0	0	
200	10	10	10	8	38	0%	100%	0%	0	0	0	0	
250	10	10	10	8	38	0%	100%	0%	0	0	0	0	
300	10	10	10	8	38	0%	100%	0%	0	0	0	0	

350	10	10	10	8	38	0%	100%	0%	0	0	0	0	
400	10	10	10	8	38	0%	100%	0%	0	0	5	5	3
450	10	10	10	8	38	0%	100%	0%	0	0	10	10	4
500	10	10	10	8	38	0%	100%	0%	0	0	20	20	4

Scenario 1: 10-10-10-6 formation with variations in the number of training between 100 to 500 batches.

This scenario uses an HR formation with 10 people in each of Divisions 1, 2, and 3, and 6 people in Division 4. This configuration was previously considered quite efficient and feasible for the number of training of up to 250 batches based on the results of HR sensitivity analysis. In this scenario, a variation of the number of training is carried out ranging from 100 to 500 batches to test the capacity limit of the formation in maintaining workload balance and minimizing constraint violations. The main outcomes of this scenario are:

1. Although the number of training has increased to 500 batches, the distribution of workloads remains in the normal category of 100%, demonstrating the resilience of the model in distributing tasks fairly despite relatively limited human resources.
2. When the number of training reaches 300 batches, violations of constraints appear, especially at the maximum limit of training per individual, namely the number of training 300: 5 violations, the number of training 400: 15 violations and the number of training 500: 30 violations. This showed that a formation with only 6 men in Division 4 was starting to become overwhelmed with handling the growing training load.
3. There was an increase in computing time from less than 1 hour (for 100–300 trainings) to 6 hours on 500 trainings, indicating increased processing complexity.

Scenario 2: 10-10-10-8 formation with a variation in the number of training between 100 to 500 batches.

This scenario is a development of the previous scenario, where the number of human resources in Division 4 (finance) was increased to 8 people. This addition is based on real conditions in 2024, where there is an additional allocation of 2 people in the Finance Division. The goal is to overcome the limited capacity of the division, which in the previous scenario was the main source of violations due to overcrowded assignments. Similar to Scenario 1, the number of training is varied from 100 to 500 batches to evaluate the extent to which these formations are able to maintain the stability of the scheduling system and minimize constraint violations. The main outcomes of this scenario are:

1. The addition of human resources in Division 4 (from 6 to 8 people) has a significant impact on maintaining a balanced work distribution even up to 500 trainings.
2. Although the number of trainings increased, the number of violations remained much less than in Scenario 1, which was 400 trainings: only 5 violations and 500 training numbers: 20 violations. This proves that adding human resources in the division that was previously a "bottleneck" is very effective in reducing violations.
3. The processing time is relatively lower than scenario 1, with a maximum time of 4 hours for 500 trainings. This shows that feasible solutions can be found faster with more adequate HR formation.

Overall, in the sensitivity analysis of the number of trainings, it was concluded that:

1. Scenario 1 shows limitations in handling large amounts of training, mainly due to the limitations of human resources in Division 4. Violations increased significantly at 400–500 trainings, exceeding the established epsilon limit (10).
2. Scenario 2 is much more stable and optimal, with a ≤ 20 violation for the number of trainings reaching 500. This reinforces the previous conclusion that the increase in HR in Division 4 significantly improves the flexibility of the system and the quality of scheduling solutions.
3. The 10-10-10-8 formation is more recommended for long-term and large training scenarios because it has better performance in terms of the number of breaches, workload distribution, and compute efficiency.

CONCLUSIONS

Based on the results of model development, model completion and result analysis, it can be concluded that this study has succeeded in answering five main objectives systematically. The conclusions of this study are described as follows: This study succeeded in building a mathematical model for scheduling work teams at government training institutions, namely the Yogyakarta Industrial Training Center by considering important factors such as the availability of human resources, the number of trainings, location, and duration. This model is able to represent real conditions well and can be used as a basis for systematically arranging schedules. The results of the application of the model show its effectiveness in reducing the inequality of workload distribution between personnel. Workload deviations have been minimized, with the proportion of human resources in the normal workload category increasing significantly compared to conventional approaches. The model is able to produce a more optimal scheduling solution in terms of HR allocation efficiency and minimization of constraint violations. This supports faster and more precise decision-making in complex scheduling situations. Comparison of optimization results with conventional scheduling shows the advantages of the model in terms of equal distribution of workload, reduction of schedule conflicts, and increased utilization of human resources. In addition, the time to prepare schedules becomes more efficient and well documented. Based on the results of the case study, it is recommended to develop and implement this automatic scheduling system based on this mathematical model in the Training Center and other training institutions. The implementation of this system is expected to be able to improve the quality of human resource management and overall operational effectiveness. Based on the findings and results of the implementation of the model in this case study, there are several suggestions that can be given both for practical application of the model in the field and for the development of research in the future.

1. Model Implementation: The scheduling model that has been developed should be implemented gradually in the Training Center, accompanied by technical training for operators and integrated with the institution's information system.
2. System Development: The model needs to be developed with flexible features, a user-friendly interface, and consideration of cost aspects and personnel preferences. This will increase efficiency while expanding its application.
3. Further Research: It is recommended to test other optimization methods such as the hybrid approach, as well as extend the study to other training institutions to test the generality of the model. Sensitivity analysis and validation with stakeholders are also important for refinement.

REFERENCES

Abdipoor, S., Yaakob, R., Goh, S. L., & Abdullah, S. (2023). Meta-heuristic approaches for the University Course Timetabling Problem. In Intelligent Systems with Applications (Vol. 19). Elsevier B.V. https://doi.org/10.1016/j.iswa.2023.200253

Adawiyah, W. (2013). Analysis of Human Resources Workload in Lettuce Vegetable Commodity Production Activities (Case Study: CV Spirit Wira Utama). Thesis at the Faculty of Economics and Management, Bogor Agricultural University, Bogor.

Al-Yakoob, S. M., & Sherali, H. D. (2015). Mathematical models and algorithms for a high school timetabling problem. Computers and Operations Research, 61, 56–68. https://doi.org/10.1016/j.cor.2015.02.011

Amrulloh, A., & Sela, E. I. (2021). Course scheduling optimization using genetic algorithm and tabu search. Journal of Computer Technology and Systems, 9(3), 157–166. https://doi.org/10.14710/jtsiskom.2021.14137

Assi, M., Halawi, B., & Haraty, R. A. (2018). Genetic Algorithm Analysis using the Graph Coloring Method for Solving the University Timetable Problem. Procedia Computer Science, 126, 899–906. https://doi.org/10.1016/j.procS.2018.08.024

Badoni, R. P., Gupta, D. K., & Mishra, P. (2014). A new hybrid algorithm for university course timetabling problem using events based on groupings of students. Computers and Industrial Engineering, 78, 12–25. https://doi.org/10.1016/j.cie.2014.09.020

Bakhshandeh, A., & Mirzapour Al-e-Hashem, S. M. J. (2024). A multi-objective scheduling model in medical tourism centers considering multi-task staff training. Engineering Applications of Artificial Intelligence, 131. https://doi.org/10.1016/j.engappai.2023.107808

Yogyakarta Industrial Training Center, (2023). Monitoring and Evaluation Report, Yogyakarta: BDI Yogyakarta.

Battistutta, Schaerf, M. & Urli, T., (2017). Feature-based tuning of single-stage simulated annealing for examination timetabling. Annals of Operations Research, Volume 252 (2), pp. 239-254.

Busetti, F., (2003). Simulated Annealing overview, s.l.: www.geocities.com/francorbusetti/saweb..

Carlsson, M., Ceschia, S., Di Gaspero, L., Mikkelsen, R. Ø., Schaerf, A., & Stidsen, T. J. R. (2023). Exact and metaheuristic methods for a real-world examination timetabling problem. Journal of Scheduling, 26(4), 353–367. https://doi.org/10.1007/s10951-023-00778-6

Ceschia, S., Di Gaspero, L., & Schaerf, A. (2023). Educational timetabling: Problems, benchmarks, and state-of-the-art results. In European Journal of Operational Research (Vol. 308, Issue 1, pp. 1–18). Elsevier B.V. https://doi.org/10.1016/j.ejor.2022.07.011

Chandra, L., (2016). Application of the Taboo Search Algorithm for Subject Scheduling at Private Vocational School Pelita-2 Aekkanopan. Journal of Computer Research (JURIKOM), Volume 3, No. 6, pp. 74-79.

Chen, M., Tang, X., Song, T., Wu, C., Liu, S., & Peng, X. (2020). A Tabu search algorithm with controlled randomization for constructing feasible university course timetables. Computers and Operations Research, 123. https://doi.org/10.1016/j.cor.2020.105007

Colajanni, G., & Daniele, P. (2021). A new model for curriculum-based university course timetabling. Optimization Letters, 15(5), 1601–1616. https://doi.org/10.1007/s11590-020-01588-x

Dewi, U. & Satrya, A. (2012). Analysis of Labor Needs Based on Employee Workload at PT PLN (Persero) Distribution of Greater Jakarta and Tangerang in the Field of Human Resources and Organization. Department of Human Resources Management, Faculty of Economics, University of Indonesia, Depok.

Dimitsas, A., & Gogos, C. (2024). Finding Near Optimal Solutions to the Thesis Defense Timetabling Problem by Exploiting Symmetries. Operations Research Forum, 5(3). https://doi.org/10.1007/s43069-024-00346-4

Dunke, F., & Nickel, S. (2023). A matheuristic for customized multi-level multi-criteria university timetabling. Annals of Operations Research, 328(2), 1313–1348. https://doi.org/10.1007/s10479-023-05325-2

Filippini, F., Ardagna, D., Lattuada, M., Amaldi, E., Riedl, M., Materka, K., Skrzypek, P., Ciavotta, M., Magugliani, F., & Cicala, M. (2021). ANDREAS: Artificial intelligence traiNing scheDuler for accElerAted resource clusterS. Proceedings - 2021 International Conference on Future Internet of Things and Cloud, FiCloud 2021, 388–393. https://doi.org/10.1109/FiCloud49777.2021.00063

Giordano, G. et al., (2020). Nat. Med, pp. 1-6.

Griffiths, J. P., Kerr, C., Waites, W. & Stuart, R., (2021). Mathematical modeling as a tool for policy decision making: Applications to the COVID-19 pandemic. In: Network models and COVID-19 modeling. s.l.:s.n., p. 291.

Hafsa, M., Wattebled, P., Jacques, J., & Jourdan, L. (2025). Solving a multiobjective professional timetabling problem using evolutionary algorithms at Mandarine Academy. International Transactions in Operational Research, 32(1), 244–269. https://doi.org/10.1111/itor.13276

Hartadi, R., Hidayat, A. & Utomo, V., (2016). Course Scheduling Application Design (Case Study: STMIK Semarang Provisi. Ferris wheel Inform, Volume 4, No.1, pp. 31-40.

Haryadi, D. & Jamal, A., (2015). Lecturer Preferences in the Lecture Scheduling Process Using Genetic Algorithms Case Study: Al Azhar University Indonesia. Al-Azhar Indonesia Journal Science and Technology Series, Volume 3, No. 2.

Havås, J., Olsson, A., Mirjam, J. P., & Schierscher, S. (2013). Modeling and optimization of university timetabling A case study in integer programming.

Hidayatullah, A. & Santosa, B., (2014). Optimization of Nurse Scheduling Problem (Case Study of Dr. Soetomo Surabaya Hospital). Engineering Journal.

Hossain, S. I. et al., (2019). Optimization of University Course Scheduling Problem using Particle Swarm Optimization with Selective Search. Expert Systems With Applications, Volume 127, pp. 9-24.

Imran Hossain, S., Akhand, M. A. H., Shuvo, M. I. R., Siddique, N., & Adeli, H. (2019). Optimization of University Course Scheduling Problem using Particle Swarm Optimization with Selective Search. Expert Systems with Applications, 127, 9–24. https://doi.org/10.1016/j.eswa.2019.02.026

Julianto, V., (2017). Multiobjective Model Optimization using Combination of Bat Algorithm with Differential Evolution (DE) and Levy Flight Trajectory. Proceedings of the Seminar on Research Results, pp. 159-164.

Kilincli Taskiran, G., & Zhang, X. (2017). Mathematical models and solution approach for cross-training staff scheduling at call centers. Computers and Operations Research, 87, 258–269. https://doi.org/10.1016/j.cor.2016.07.001

Lee, S.-Y., Kim, Y.-D., & Lee, H.-J. (2009). Heuristic Algorithm for a Military Training Timetabling Problem. In Asia Pacific Management Review (Vol. 14, Issue 3). www.apmr.management.ncku.edu.tw

Mahrijal, Zulvan., Sumarsa, Amar., & Widyastiti, Maya. (2024). Optimization of Subject Scheduling using the Integer Linear Programming Method (Case Study: SMA – Al – Hikmah). Scientific Journal of Mathematics Volume 4, No. 1, March 2024, pp. 22-32, E-ISSN: 2809-3755

Mallari, C. B., San Juan, J. L., & Li, R. (2023). The university coursework timetabling problem: An optimization approach to synchronizing course calendars. Computers and Industrial Engineering, 184. https://doi.org/10.1016/j.cie.2023.109561

Marler, R. T., & Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. Structural and Multidisciplinary Optimization, 26(6), 369–395.

Mokhtari, M., Vaziri Sarashk, M., Asadpour, M., Saeidi, N., & Boyer, O. (2021). Developing a Model for the University Course Timetabling Problem: A Case Study. Complexity, 2021. https://doi.org/10.1155/2021/9940866

Novera, W. (2012). Analysis of Workload and Employee Needs of the Academic and Student Administration Section (Case Study of the Administrative Unit of the Department at Bogor Agricultural University). Thesis at the Faculty of Economics and Management, Bogor Agricultural University, Bogor.

Nugraha, F., (2020). Education and Training: Concepts and Implementation in Human Resource Development. 1 ed. Jakarta: R&D Press.

Nugroho, Y. A. B., (2020). Human Resource Management. 1 ed. Jakarta: Publisher of the Catholic University of Indonesia Atma Jaya.

Priambodo, M. A., Nhita, F. & Aditsania, A., (2016). Course scheduling uses a hybrid method of genetic algorithms and ant colony algorithms. e-Proceedings of Engineering, Volume Vol.3, No 3, p. 3711.

Rahman, R. A., Wahid, J., Wahab, A. A., Hassan, S., Ahmad, R., & Ahmad, A. (2022). Systematic Literature Review of Metaheuristic Methodologies for High School Timetabling Problem. Proceedings - AiIC 2022: 2022 Applied Informatics International Conference: Digital Innovation in Applied Informatics during the Pandemic, 190–195. https://doi.org/10.1109/AiIC54368.2022.9914034

Rappos, E., Thiémard, E., Robert, S., & Hêche, J. F. (2022). A mixed-integer programming approach for solving university course timetabling problems. Journal of Scheduling, 25(4), 391–404. https://doi.org/10.1007/s10951-021-00715-5

Rezaei, M., Shamsaei, F., Mohammadian, I., & Van Vyve, M. (2012). A heuristic method to schedule training programs for Small and Medium Enterprises. European Journal of Operational Research, 217(3), 600–608. https://doi.org/10.1016/j.ejor.2011.09.005

Ribic et al. (2017). Modelling Constraints in School Timetabling using Integer Linear Programming. XXV International Conference on Information, Communication and Automation Technologies (ICAT). DOI:10.1109/ICAT.2015.7340525

Rosyidi, C. N., Budiningsih, E., & Jauhari, W. A. (2019). Scheduling of undergraduate thesis examination: a case study in Industrial Engineering Department of Universitas Sebelas Maret. Journal of Industrial Engineering International, 15, 209–221. https://doi.org/10.1007/s40092-019-00330-2

Santoso, B. & Ai, T. J., (2017). Introduction to Metaheuristics. First ed. Surabaya: ITS Techno Science.

Saviniec, L., Santos, M. O., & Costa, A. M. (2018). Parallel local search algorithms for high school timetabling problems. European Journal of Operational Research, 265(1), 81–98. https://doi.org/10.1016/j.ejor.2017.07.029

Saviniec, L., Santos, M. O., Costa, A. M., & Santos, L. M. R. dos. (2020). Pattern-based models and a cooperative parallel metaheuristic for high school timetabling problems. European Journal of Operational Research, 280(3), 1064–1081. https://doi.org/10.1016/j.ejor.2019.08.001

Skoullis, V. I., Tassopoulos, I. X., & Beligiannis, G. N. (2017). Solving the high school timetabling problem using a hybrid cat swarm optimization based algorithm. Applied Soft Computing Journal, 52, 277–289. https://doi.org/10.1016/j.asoc.2016.10.038

Sylejmani, K., Gashi, E., & Ymeri, A. (2023). Simulated annealing with penalization for university course timetabling. Journal of Scheduling, 26(5), 497–517. https://doi.org/10.1007/s10951-022-00747-5

Taskiran & Kilincli, G., (2015). Mathematical Models and Solution Approach for Staff Scheduling with Cross-Training at Call Centers, Wright State University: Browse all Theses and Dissertations.

Tauhid, M. F., Nurcahyo, R., & Farizal, D. (2019). IDEC National Seminar and Conference on Optimization of Leadership Class Scheduling in Non-University Institutions Using Genetic Algorithms.

T. Tridoyo, and S. Sriyanto, (2014). Workload Analysis with Full Time Equivalent Method to Optimize Employee Performance at PT Astra International Tbk-Honda Sales Operation Region Semarang, Industrial Engineering Online Journal, vol. 3, no. 2, Mar. 2014.

Urva, G., & Sellyana, A. (2019). Genetic algorithm for optimization of lecturer schedule preparation. Journal of Physics: Conference Series, 1175(1). https://doi.org/10.1088/1742-6596/1175/1/012042

Via, Y. V., Munir, M. S. & Muhammad, A., (2017). Hospital Employee Scheduling Optimization using Genetic Algorithms. Journal of Scheduling, Volume XII No 1.

Wahyuningsih, D., & Helmud, E. (2020). Application of Genetic Algorithms for Scheduling Optimization at MTS Negeri 1 Pangkalpinang. Journal of Sisfokom (Information Systems and Computers), 9(3), 435–441. https://doi.org/10.32736/sisfokom.v9i3.994

Waluyo, A. & Fatich, E., (2017). Website Design of the Youth and Sports Education Office (Case Study of the Youth and Sports Education Office of Kebumen Regency. J-SAKTI (Journal of Science, Volume I (2), p. 186.

Williams, H., (2013). Model Building in Mathematical Programming. s.l.:John Wiley & Sons.

Wirawan, I. M. A., (2017). Reasoning Methods in Artificial Intelligence. First ed. Depok: Rajawali Pers.

Xiang, K., Hu, X., Yu, M., & Wang, X. (2024). Exact and heuristic methods for a university course scheduling problem. Expert Systems with Applications, 248. https://doi.org/10.1016/j.eswa.2024.123383

Yang, X., (2010). A new Metaheuristic Bat-Inspired Algorithm: Nature Inspired Cooperative Strategies for Optimization. Studies in Computational Berlin, Volume 284, pp. 65-74.